Chapter 1.1, Question 2a
Solving $z^2 + 36 = 0$ we find that

$$z = \frac{\pm\sqrt{(-1)(4)(36)}}{2} = \pm 6i$$

Chapter 1.1, Question 2b
Solving $2z^2 + 2z + 5 = 0$ via the quadratic formula, we get:

$$z = \frac{-2 \pm \sqrt{4 - 4(2)(5)}}{4} = \frac{2 \pm 6i}{4} = \frac{1 \pm 3}{2}$$

Chapter 1.1, Question 3b
Given that $|w - 2| = 1$ and letting $w = x + yi$, we find that this implies that $\sqrt{x^2 + y^2} = 1$. Squaring both sides, $x^2 + y^2 = 1$, and so the set of elements of $\mathbb{C}$ satisfying this equation is the circle of radius 1 around 2.

Chapter 1.1, Question 3d
Given the equation, $|w + 2| = |w - 2|$, we will square both sides and solve:

$$|w + 2|^2 = |w - 2|^2$$

$$(Re(w) + 2)^2 + Im(w)^2 = (Re(w) - 2)^2 + Im(w)^2$$

Solving that equation, we find that

$$4Re(w) = -4Re(w)$$

And so the solutions lie on the line $Re(w) = 0$, that is, the locus of solutions is the imaginary axis.

Chapter 1.1, Question 3f
Let $z = a + bi$.

$$Re((1 - i)\bar{z}) = Re((1 - i)(a - bi)) = Re(a + bi - ai - b) = a - b$$

Now, setting that equal to zero, we get

$$a - b = 0$$

So, all solutions lie on the line $Re(z) = Im(z)$. Note that this is precisely the line $y = x$ in $\mathbb{R}^2$.

Chapter 1.1, Question 5a
Given $-1+i = z$, first note that $r = |z| = \sqrt{2}$. Then, to find θ, we recall that $Re(z) = r\cos\theta$:

$$-1 = r\cos\theta$$

$$-1 = \sqrt{2}\cos\theta$$

$$\cos\theta = -\frac{\sqrt{2}}{2}$$

Also, $Im(z) = r\sin\theta$

$$1 = r\sin\theta$$

$$1 = \sqrt{2}\sin\theta$$

$$\sin\theta = \frac{\sqrt{2}}{2}$$

So, we want angles θ such that $\sin\theta = \frac{\sqrt{2}}{2}$ and $\cos\theta = -\frac{\sqrt{2}}{2}$, and so,

$$\theta = \frac{3\pi}{4} + 2\pi k \ , \ k \in \mathbb{Z}$$

And the final form of the solution should be

$$z = \sqrt{2}\cos\left(\frac{3\pi}{4} + 2\pi k\right) + i\sqrt{2}\sin\left(\frac{3\pi}{4} + 2\pi k\right)$$

Chapter 1.1, Question 5b Given $z = 1 + i\sqrt{3}$, we proceed as is part (a): $|z| = \sqrt{1+3} = 2$,

$$r\cos\theta = 1$$

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

Solving for $\sin\theta$:

$$2\sin\theta = \sqrt{3}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

And again recalling basic trig,

$$\theta = \frac{\pi}{3} + 2\pi k \ , \ k \in \mathbb{Z}$$

So, the final form of our solution is

$$z = 2\cos\left(\frac{\pi}{3} + 2\pi k\right) + 2i\sin\left(\frac{\pi}{3} + 2\pi k\right)$$

Chapter 1.1, Question 6

Part (a): $r\cos\theta + ir\sin\theta = \sqrt{3}\left(\frac{\sqrt{2}}{2}\right) + i\sqrt{3}\left(\frac{\sqrt{2}}{2}\right)$

Part (c): $r\cos\theta + ir\sin\theta = 4(0) + i(4)(-1) = -4i$

Part (f): $r\cos\theta + ir\sin\theta = \sqrt{2}\cos\left(\frac{9\pi}{4}\right) + i\sqrt{2}\sin\left(\frac{9\pi}{4}\right) = \sqrt{2}\frac{\sqrt{2}}{2} + i\sqrt{2}\frac{\sqrt{2}}{2} = 1 + i$

Chapter 1.2, Question 23 Given $z^5 = i$, we may rewrite i as $1 \cdot \left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$. Then, we have the new equation

$$z^5 = 1 \cdot \left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$$

Using DeMoivre's formula to solve, we get

$$z = \left(\cos\left(\frac{\pi}{10} + \frac{2\pi k}{5}\right) + i\sin\left(\frac{\pi}{10} + \frac{2\pi k}{5}\right)\right) \ , k \in \{0, 1, 2, 3, 4\}$$

Chapter 1.2, Question 24

Given $(z+1)^4 = 1 - i$, we may rewrite $1 - i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$ Using DeMoivre's formula to solve, we get

$$z + 1 = 2^{\frac{1}{8}}\left(\cos\left(-\frac{\pi}{16} + \frac{2\pi k}{4}\right) + i\sin\left(-\frac{\pi}{16} + \frac{2\pi k}{4}\right)\right) \ , k \in \{0, 1, 2, 3\}$$

So finally solving for z, we have

$$z = 2^{\frac{1}{8}}\left(\cos\left(-\frac{\pi}{16} + \frac{2\pi k}{4}\right) - 2^{-\frac{1}{8}} + i\sin\left(-\frac{\pi}{16} + \frac{2\pi k}{4}\right)\right)$$

Chapter 1.2, Question 25

$z^8 = -1$. We write $-1 = \cos(\pi) + i\sin(\pi)$, and we find

$$z = \cos\left(\frac{\pi}{8} + \frac{2\pi k}{8}\right) + i\sin\left(\frac{\pi}{8} + \frac{2\pi k}{8}\right) \ , k \in \{0, 1, ..., 7\}$$

Chapter 1.2, Question 26

$z^3 = 8$. We write $8 = 8(\cos(0) + i\sin(0))$. Then,

$$z = 2\left(\cos\left(\frac{2\pi k}{3}\right) + i\sin\left(\frac{2\pi k}{3}\right)\right) \ , k \in \{0, 1, 2\}$$

Chapter 1.3, Question 2

$B = \{z : |z| < 1 \text{ or } |z-3| \leq 1\}$ is the union of two discs of radius 1, one open around 0 and one closed around 3.

Part (a): $\text{Int}(B) = \{z : |z| < 1 \text{ or } |z-3| < 1\}$ and $\partial B = \{z : |z| = 1 \text{ or } |z-3| = 1\}$

Part (b): B is neither open nor closed, since it contains only part of its boundary. (Recall that it would be open if it contained none of its boundary, and closed if it contained all of its boundary.)

Part (c): The interior of B is not (path) connected, as there is not path from 0 to 3. (Note that it is in fact also not connected in the standard topological sense.)

Chapter 1.3, Question 4

$D = \{z : Re(z^2) = 4\}$. Let $z = a + bi$. Then, $Re(z^2) = Re(a^2 + 2abi - b^2) = a^2 - b^2$. So, $D = \{a + bi : a^2 - b^2 = 4\}$. Thus, D is the graph of a hyperbola.

Part (a): $\text{Int}(D) = \emptyset$ and $\partial D = D$.

Part (b): D is closed since $\partial D = D \subseteq D$.

Part (c): D has empty interior, so it is vacuously connected.

Chapter 1.3, Question 8

$H = \{z = x + iy : -\pi \leq y < \pi\}$ is a horizontal strip from $-\pi$ to π.

Part (a): $\text{Int}(H) = \{z = x + iy : -\pi < y < \pi\}$ and $\partial H = \{z = x + iy : y = -\pi \text{ or } y = \pi\}$

Part (b): H is neither open nor closed, since it contains only part of its boundary.

Part (c): $\text{Int}(H)$ is connected.